INVERSE OF A FUNCTION DEFINED BY ORDERED PAIRS

If $f(x)$ is a one-to-one function whose ordered pairs are of the form (x, y), then its inverse function $f^{-1}(x)$ is the set of ordered pairs (y, x).

Switch x + y

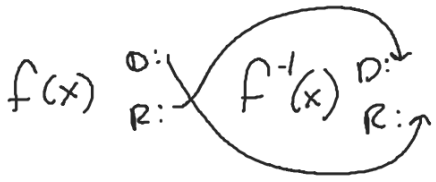
f^{-1} → Inverse

Find the inverse of the function $\{(0, 3), (1, 5), (2, 7), (3, 9)\}$. Determine the domain and range of the inverse function.

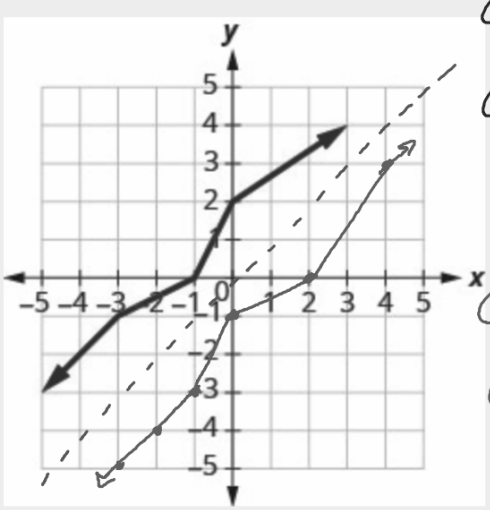
$f^{-1}\{(3,0), (5,1) (7,2) (9,3)\}$

D: $\{3, 5, 7, 9\}$

R: $\{0, 1, 2, 3\}$



Graph, on the same coordinate system, the inverse of the one-to one function shown.



(0,2) (-1,0) (-3,-1) (3,4) (-4,-2)

(-5,-3)

Inverse

(2,0) (0,-1) (-1,-3) (4,3) (-2,-4)

(-3,-5)

INVERSE FUNCTIONS

$$f^{-1}(f(x)) = x, \text{ for all } x \text{ in the domain of } f$$
$$f\left(f^{-1}(x)\right) = x, \text{ for all } x \text{ in the domain of } f^{-1}$$

$(f \circ g)(x) = (g \circ f)(x)$

$\times$

Verify that $f(x) = 5x - 1$ and $g(x) = \frac{x+1}{5}$ are inverse functions.

$(f \circ g)(x) = f(g(x)) = 5x - 1$

$f\left(\frac{x+1}{5}\right) = 5\left(\frac{x+1}{5}\right) - 1$

$x + 1 - 1$

x

$(g \circ f)(x) = g(f(x))$

$g(5x-1) = \frac{5x-1+1}{5}$

$= \frac{5x}{5}$

$= x$

Verify that the functions are inverse functions.

$f(x) = 2x + 6$ and $g(x) = \frac{x-6}{2}$.

$f(g(x)) = 2x+6$

$f\left(\frac{x-6}{2}\right) = 2\left(\frac{x-6}{2}\right)+6$

$x-6+6$

x

$g(f(x)) = \frac{x-6}{2}$

$g(2x+6) = \frac{2x+6-6}{2}$

$= \frac{2x}{2}$

$= x$

How to Find the inverse of a One-to-One Function

Find the inverse of $f(x) = 4x + 7$.

Switch x + y and solve for y

$y = 4x + 7$

$x = 4y + 7$

$-7 \qquad -7$

$\frac{x-7}{4} = \frac{4y}{4}$

$f^{-1}(x) = \frac{x-7}{4}$

Find the inverse of the function $f(x) = 5x - 3$.

$y = 5x - 3$

$x = 5y - 3$

$x + 3 = 5y$

$f^{-1}(x) = \frac{x+3}{5}$

$\frac{1}{5}x + \frac{3}{5}$

Find the inverse of $f(x) = \sqrt[5]{2x-3}$.

$$y = \sqrt[5]{2x-3}$$

$$(x)^5 = \left(\sqrt[5]{2y-3}\right)^5$$

$$x^5 = 2y-3$$

$$x^5+3 = 2y$$

$$f^{-1}(x) = \frac{x^5+3}{2}$$

Find the inverse of the function $f(x) = \sqrt[4]{6x - 7}$.

$f^{-1}(x) = \frac{x^4 + 7}{6}$

1, 3, 5, 9, 11
13, 17-20
21, 23, 27, 29
31-37 odd
53-61 odd